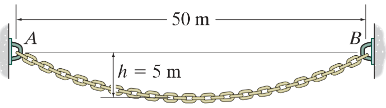
7–107. If $h = 5$ m, determine the maximum tension developed in the chain and its length. The chain has a mass per unit length of 8 kg/m.



As shown in Fig. *a*, the origin of the *x*, *y* coordinate system is set at the lowest point of the cable. Here, $w(s) = 8(9.81)\,\text{N/m} = 78.48\,\text{N/m}$.

$$\frac{d^2y}{dx^2} = \frac{78.48}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

If we set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$, then

$$\frac{du}{\sqrt{1+u^2}} = \frac{78.48}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{78.48}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{78.48}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{78.48}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{78.48}{F_H}x} - e^{-\frac{78.48}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{78.48}{F_H}x \qquad (1)$$

Integrating Eq. (1),

$$y = \frac{F_H}{78.48}\cosh\left(\frac{78.48}{F_H}x\right) + C_2$$

Applying the boundary equation $y = 5$ m at $x = 25$ m,

$$5 = \frac{F_H}{78.48}\left\{\cosh\left(\frac{78.48}{F_H}(25)\right) - 1\right\}.$$

Solving by trial and error,

$$F_H = 4969.06\,\text{N}$$

The maximum tension occurs at either points A or B where the chain makes the greatest angle with the horizontal. Here,

$$\theta_{max} = \tan^{-1}\left(\frac{dy}{dx}\bigg|_{x=25\text{ m}}\right) = \tan^{-1}\left\{\sinh\left(\frac{78.48}{F_H}(25)\right)\right\} = 22.06°$$

Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{4969.06}{\cos 22.06°} = 5361.46\text{ N} = 5.36\text{ kN} \qquad \textbf{Ans.}$$

Referring to the free-body diagram shown in Fig. b,

$+\uparrow \Sigma F_y = 0; \qquad T\sin\theta - 8(9.81)s = 0$

$+\rightarrow \Sigma F_y = 0; \qquad T\cos\theta - 4969.06 = 0$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = 0.015794s$$

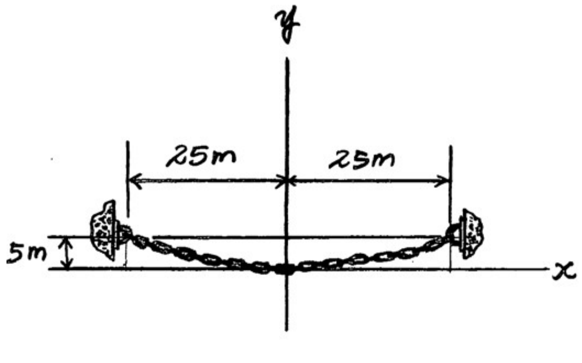
Equating Eqs. (1) and (2),

$$\sinh\left[\frac{78.48}{4969.06}x\right] = 0.015794s$$

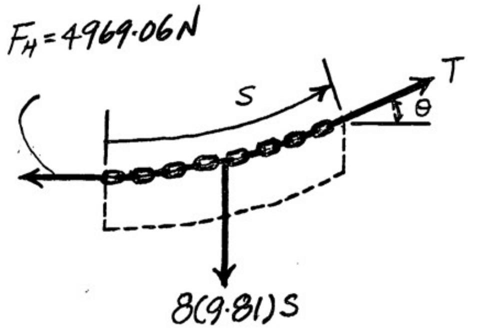
$$s = 63.32\sinh[0.01579x]$$

Thus, the length of the chain is

$$L = 2\{63.32\sinh[0.01579(25)]\} = 51.3\text{m} \qquad \textbf{Ans.}$$



(a)



(b)