Finding Distance of a Point

Plot

$$Dist2(f(2), A, P, t, N) :=$$

$$\text{Clear}(\tau, x, \kappa)\ c := [1..2]\ U := \text{matrix}(0, 2)$$

$$G(x) := \begin{bmatrix} \frac{d}{dx_1} f(x_1, x_2) \\ \frac{d}{dx_2} f(x_1, x_2) \end{bmatrix} \quad \Delta(\tau, x) := \begin{bmatrix} -\frac{d}{dx_2} f(x_1, x_2) \\ \frac{d}{dx_1} f(x_1, x_2) \end{bmatrix}$$

$$L(x) := \sum_{\kappa=1}^{2} \left(x_\kappa - A_\kappa\right)^2 + x_3 \cdot f(x_1, x_2)$$

$$\Phi(x) := \begin{bmatrix} \frac{d}{dx_1} L(x) & \frac{d}{dx_2} L(x) & \frac{d}{dx_3} L(x) \end{bmatrix}^T$$

$$DM(p, \tau, n) :=$$

$$\left[u := p^T \quad u_2 := \text{roots}\left(f(u_1, x), x, u_2\right) \right]$$

$$\text{eval}\left(\text{rkfixed}(u, 0, \tau, n, \Delta)_{[1..(n+1)][2..3]}\right)$$

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if rows(P)>1
  for k ∈ [1..rows(P)]
    U := eval(stack(U, DM(row(P, k), t_k, N_k)))
else
  U := DM(P, t, N)
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$$\left[\nu := [1..\text{rows}(U)]\ \ \Delta_\nu := \text{norme}\left(A - U_{\nu\, c}\right) \right]$$

$$a := \text{findrows}\left(\text{eval}(\text{augment}(U, \Delta)), \text{eval}(\min(\Delta)), 3\right)_{1\, c}$$

$$a := \text{eval}\left(\text{al\_nleqsolve}\left(\text{stack}\left(a^T, 0\right), \Phi(x)\right)_c^T\right)$$

$$\text{eval}\left(\left[a\ \ \delta := \text{norme}(A - a)\ \ \text{stack}\left(a, a + \frac{\delta}{2} \cdot \frac{G(a)}{\text{norme}(G(a))}^T\right) U \right]\right)$$

2D Examples

Example 1

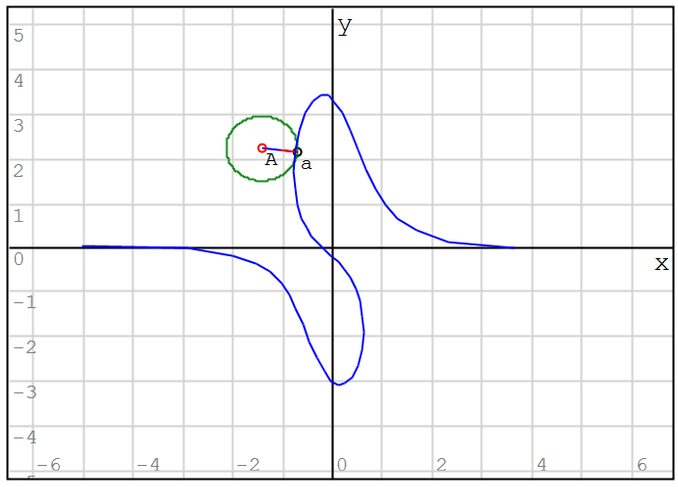
$$t_o := \text{time}(0)$$

$$\begin{cases} f(x,y) := x^2 \cdot y - \sin(x+y) \\ A := rndBox([-5\ 5],[-4\ 4]) \\ [P := [-5\ 0]\ t := -7\ N := 50] \\ [a\ \delta\ No\ U] := Dist2(f(x,y), A, P, t, N) \end{cases}$$

$$a = [-0.683\ 2.1265]$$

$$A = [-1.3951\ 2.2063]$$

$$\delta = 0.7166$$



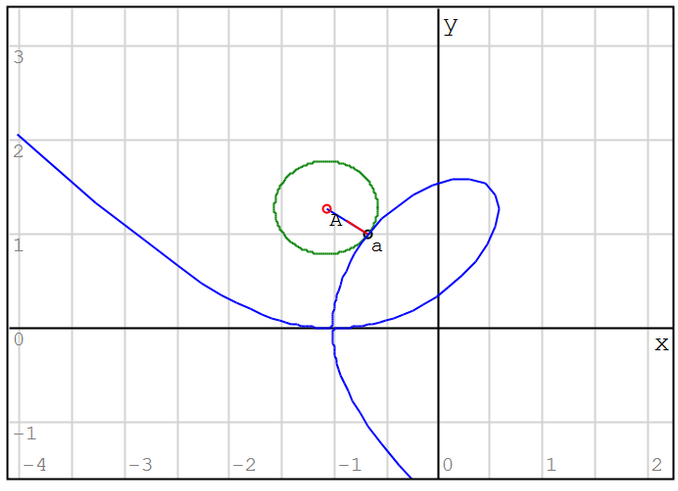
Example 2

$$\begin{cases} f(x,y) := \ln\left(\left|1+(1+x)^3-3\cdot y\cdot(1+x)+y^3\right|\right) \\ A := rndBox([-4\ 2],[-1\ 3]) \\ [P := [-4\ 2.1]\ t := -7\ N := 150] \\ [a\ \delta\ No\ U] := Dist2(f(x,y), A, P, t, N) \end{cases}$$

$$a = [-0.6632\ 0.9857]$$

$$A = [-1.0605\ 1.2749]$$

$$\delta = 0.4914$$



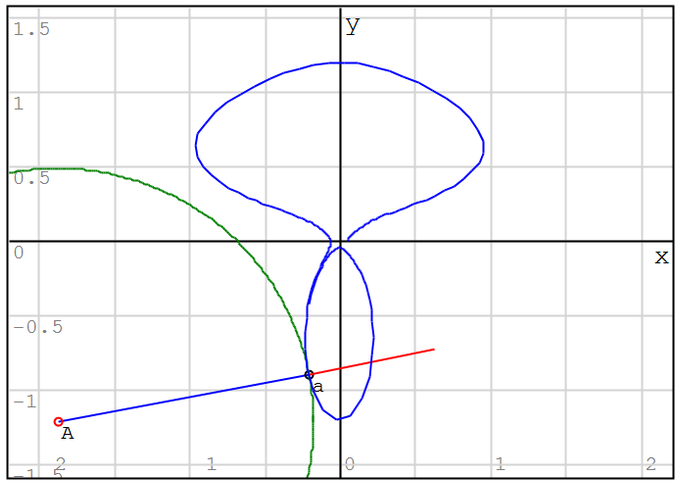
Example 3

$$\begin{cases} \alpha o := 0.842 \\ f(x,y) := \dfrac{1}{\left(x^2+y^2-\alpha o^2\right)^2+\left(2\cdot x\cdot(y-\alpha o)\right)^2}-2 \\ A := rndBox([-2\ 2],[-1.5\ 1.5]) \\ [P := [-0.2\ -0.5]\ t := 1.5\ N := 200] \\ [a\ \delta\ No\ U] := Dist2(f(x,y), A, P, t, N) \end{cases}$$

$$a = [-0.2026\ -0.8821]$$

$$A = [-1.8626\ -1.2052]$$

$$\delta = 1.6911$$



Example 4

$81 \cdot \sqrt{6 \cdot \pi} = 351.6702$

$$f(x, y) := \exp\left(-\frac{\sqrt{x^2 + y^2}}{3}\right) \cdot \frac{x^2 - 2 \cdot y^2}{351.6702} + 0.005$$

$$A := rndBox([-10\ -10], [-8\ 8])$$

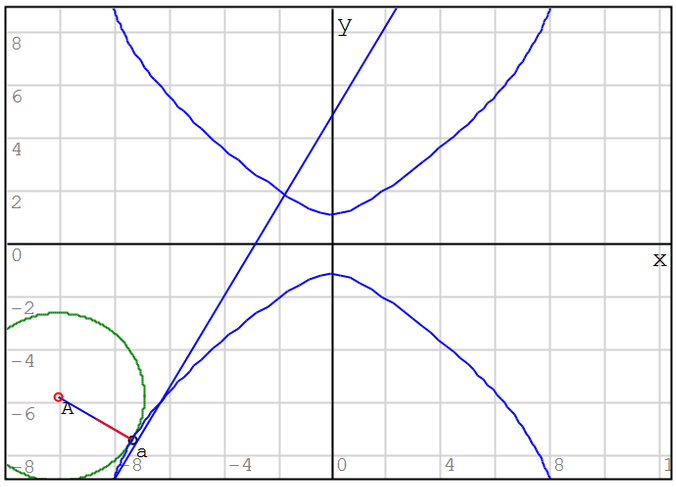
$$P := \begin{bmatrix} -8 & 8 \\ -8 & -8 \end{bmatrix} \quad t := \begin{bmatrix} 10000 \\ -10000 \end{bmatrix} \quad N := \begin{bmatrix} 200 \\ 200 \end{bmatrix}$$

$$[a\ \delta\ No\ U] := Dist2(f(x, y), A, P, t, N)$$

$a = [-7.2804\ -7.3662]$

$A = [-10\ -5.7579]$

$\delta = 3.1595$



Example 5

$$f(x, y) := 2 \cdot x^2 + 4 \cdot x \cdot y + 4 \cdot y^3 - 26 \cdot y - 22$$

$$A := rndBox([-12\ 12], [-8\ 8])$$

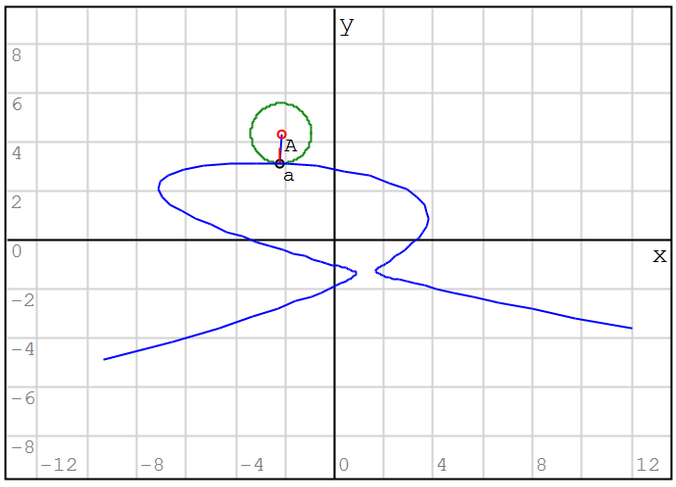
$$P := [12\ -4] \quad t := 1.5 \quad N := 100$$

$$[a\ \delta\ No\ U] := Dist2(f(x, y), A, P, t, N)$$

$a = [-2.1652\ 3.1112]$

$A = [-2.1086\ 4.329]$

$\delta = 1.2191$



Example 6

$$f(x, y) := \left(x^2 + y^2\right)^2 - \left(x^3 - 3 \cdot x \cdot y^2\right)$$

$$A := rndBox([-1\ 1], [-1\ 1])$$

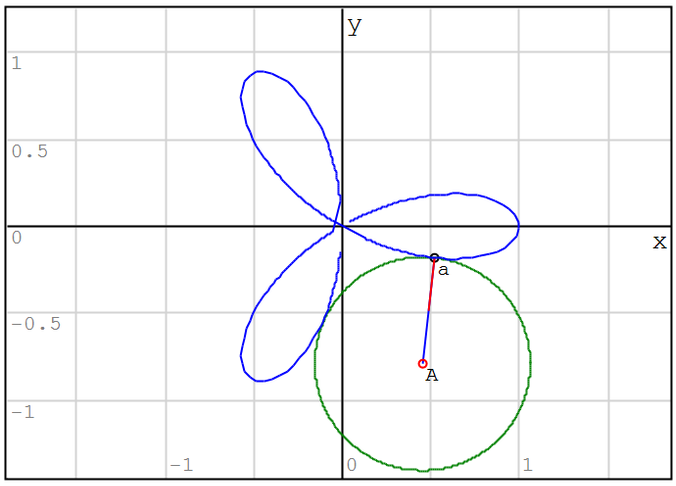
$$P := \begin{bmatrix} 0.05 & 0.05 \\ -0.05 & 0.05 \\ -0.05 & -0.05 \end{bmatrix} \quad t := \begin{bmatrix} -8 \\ -8 \\ 8 \end{bmatrix} \quad N := \begin{bmatrix} 200 \\ 200 \\ 200 \end{bmatrix}$$

$$[a\ \delta\ No\ U] := Dist2(f(x, y), A, P, t, N)$$

$a = [0.5238\ -0.1783]$

$A = [0.4571\ -0.7809]$

$\delta = 0.6063$



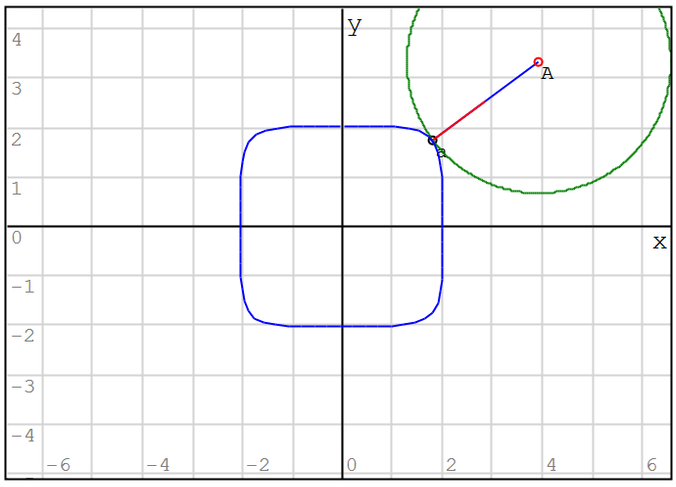
Example 7

$$f(x,y) := x^6 + y^6 - 2^6$$
$$A := rndBox([-4\ 4],[-4\ 4])$$
$$[P := [0\ 1]\ h := 0.08\ N := 60]$$
$$[a\ \delta\ No\ U] := Dist2(f(x,y), A, P, h, N)$$

$$a = [1.8299\ 1.7261]$$

$$A = [3.9372\ 3.2996]$$

$$\delta = 2.6299$$



Example 8

$$f(x,y) := (x-2)^2 + 5\cdot y^2 + x^2\cdot y^{10} - 3\cdot\sin(x\cdot y) - 9$$
$$A := rndBox([-2\ 7],[-3\ 3])$$
$$[P := [0\ 1]\ h := 1\ N := 80]$$
$$[a\ \delta\ No\ U] := Dist2(f(x,y), A, P, h, N)$$

$$a = [5.2176\ 0.0927]$$

$$A = [5.8657\ -1.2611]$$

$$\delta = 1.501$$

$$time(0) - t_o = 43.235\ s$$